

① Coterminal angles — which are same when drawn
 Ex: $361^\circ, 1^\circ$; Subtracting 360° gives them

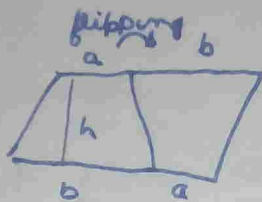
altitudes meet at orthocentre and medians meet at centroid.



If O is orthocentre, $AO = \frac{2}{3} AB$;

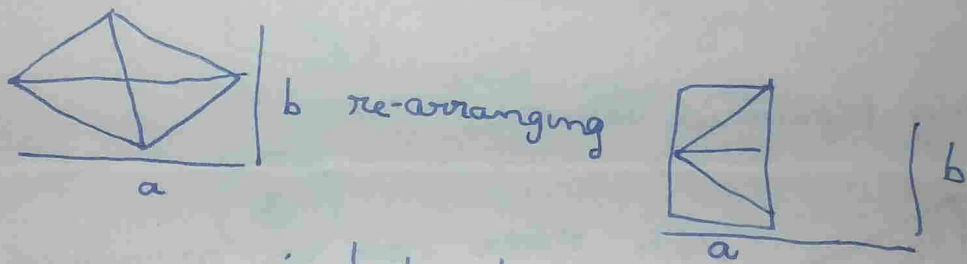
$OB = \frac{1}{3} AB$;

Trapezoid area



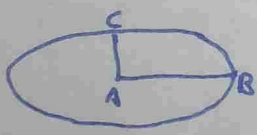
$$\therefore \frac{1}{2}(a+b)h = A$$

Rhombus area



$$\therefore \frac{1}{2}ab = \frac{1}{2}d_1d_2$$

ellipses



AB = semi major axis

AC = semi minor axis

2AB = Major axis

2AC = semi minor axis

Area can be derived by observing,



18π

from



9π

Hence

$$E_{area} = \frac{1}{2}ab$$

a = semi major axis

b = semi minor axis

Stretching x times the length/width changes area by x times

ADRIN

Cavalieri's Principle

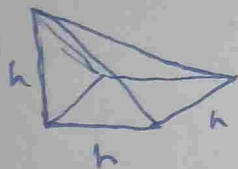
2D: If two shapes have same height and matching widths everywhere along the corresponding height, they have equal area.

3D: If two figures have same height and matching base area everywhere along the corresponding height, they have same volume.



AB is secant, touches only two points

Proving pyramid volume,



such three can be arranged to form a cube

$$\therefore 1 \text{ pyramid} = \frac{1}{3} \text{ cuboid}$$

$$\therefore P_{\text{volume}} = \frac{1}{3} A h$$

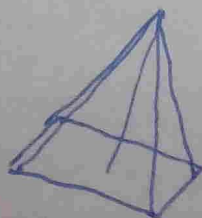
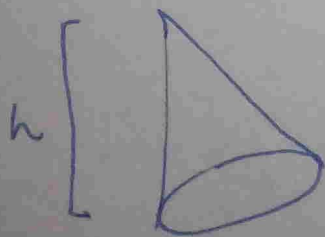
Proving cone's volumes, constructing a pyramid with same height and same base area

$$\therefore C_v = P_v \text{ Cavalieri's Principle}$$

$$P_v = \frac{1}{3} h A$$

$$\therefore C_v = \frac{1}{3} h A$$

$$= \frac{1}{3} \pi r^2 h$$



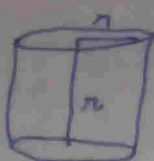
③ Proving Volume of Sphere

A cone



$$V = \frac{1}{3} \pi r^3$$

A cylinder



$$V = \pi r^3$$

Hence remaining region area =

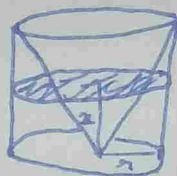
$$\frac{2}{3} \pi r^3$$



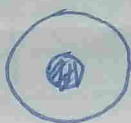
By Picking out slices



Hemisphere



our cylinder, same height and are sliced



Area

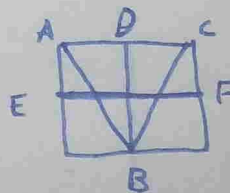
$$\text{Radius} = \sqrt{r^2 - x^2}$$

$$\text{Area} = \pi r^2 - \pi x^2$$

Outer circle = πr^2

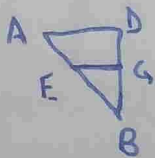
Inner circle =

Since a cross-section is like



$AD = DB = \text{radius}$

\therefore ADB is right isosceles Δ



EG is parallel to

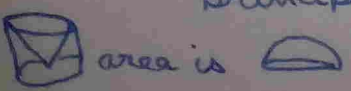
AD, \therefore Similarity of triangle is there

$$\begin{array}{l|l} AD \rightarrow DB & EG \rightarrow GB \\ r - r & x - x \end{array}$$

Hence

I.C. area = πx^2

①
②
 \therefore Any = $\pi r^2 - \pi x^2$
①, ② same
 \therefore By Cavalieri's principle

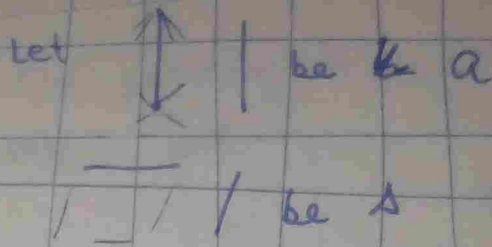
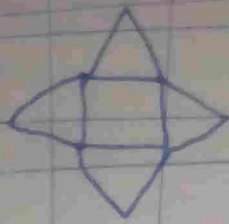
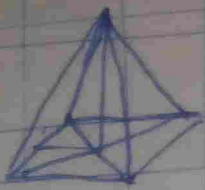


$$= \frac{2}{3} \pi r^3 \therefore \text{①} = \frac{4}{3} \pi r^3$$

ADRIN

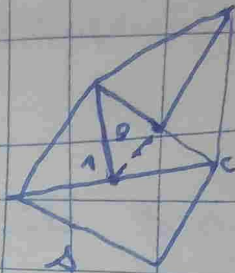
(4)

regular pyramid surface area,



$$\therefore SA = s^2 + 4\left(\frac{1}{2} b h\right)$$
$$= s^2 + 2 s h$$

and



cross-section

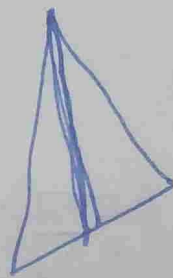
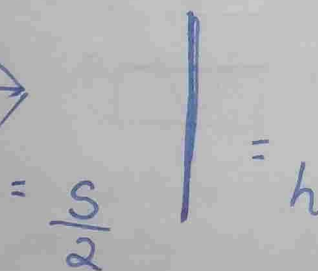
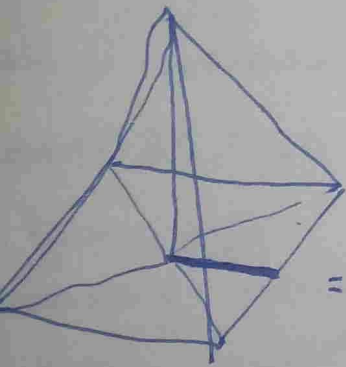
$AB = \frac{s}{2}$

$$AB^2 + \left(\frac{s}{2}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

$$AB^2 + \frac{s^2}{4} = \frac{1}{2}$$

~~$AB = \frac{s}{2}$~~

regular pyramid surface area



$$= \sqrt{\frac{s^2}{4} + h^2}$$
$$= \frac{\sqrt{s^2 + h^2}}{2}$$

$$\therefore T.S.A = s^2 + 4 \left(\frac{\sqrt{s^2 + h^2}}{2} \right)$$

$$= s^2 + 2\sqrt{s^2 + h^2}$$

Deriving cone T.S.A :



circumference = $2\pi r$
slant height = s

unrolling



Total area : πs^2

L.S.A = $\frac{2\pi r}{2\pi s}$
fraction

$\frac{TA}{TA} = \frac{\pi}{s}$

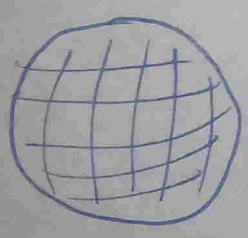
$$\therefore \text{L.S.A} = \frac{\pi}{s} \times \text{T.A} = \frac{\pi}{s} \times 2\pi s^2$$

$$= \pi r s$$

Added the base,

T.S.A of cone = $\pi r^2 + \pi r s$

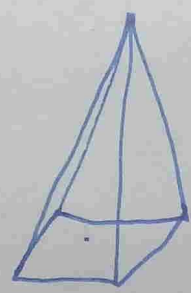
Deriving sphere T.S.A.



$V = \frac{4}{3} \pi r^3$

S.A. = A

No. of = N



pyramid base = $\frac{A}{N}$

volume = $\frac{1}{3} \times \frac{A}{N} \times r$

Since sphere is made of N pyramids

$N \times \frac{1}{3} \times \frac{A}{N} \times r = \text{Sphere volume}$

$\therefore \frac{4}{3} \pi r^3 = \frac{1}{3} A r$

$A = 4\pi r^2$

⑥ Translation: moving

$$(x, y) \rightarrow (x+3, y-1)$$

~~Reflection~~ Reflection

Point	Axis	Reflected
(a, b)	x	$(a, -b)$
(a, b)	y	$(-a, b)$
(a, b)	$y=x$	$(-a, -b)$
(a, b)	origin	(b, a)

Rotation:

$+90^\circ$ means \curvearrowright anti clockwise

$$(x, y) \curvearrowright 90^\circ \rightarrow (-y, x)$$

Geometry X

EdX

Point-slope form: $y - y_1 = m(x - x_1)$

